PHYSOR 2010 – Advances in Reactor Physics to Power the Nuclear Renaissance
Pittsburgh, Pennsylvania, USA, May 9-14, 2010, on CD-ROM, American Nuclear Society, LaGrange Park, IL (2010)

STATISTICAL IMPLICATIONS IN MONTE CARLO DEPLETIONS

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ABSTRACT

As a result of steady advances of computer power, continuous-energy Monte Carlo depletion analysis is attracting considerable attention for reactor burnup calculations. The typical Monte Carlo analysis is set up as a combination of a Monte Carlo neutron transport solver and a fuel burnup solver. Note that the burnup solver is a deterministic module. The statistical errors in Monte Carlo solutions are introduced into nuclide number densities and propagated along fuel burnup. This paper is towards the understanding of the statistical implications in Monte Carlo depletions, including both statistical bias and statistical variations in depleted fuel number densities. The deterministic Studsvik lattice physics code, CASMO-5, is modified to model the Monte Carlo depletion. The statistical bias in depleted number densities is found to be negligible compared to its statistical variations, which, in turn, demonstrates the correctness of the Monte Carlo depletion method. Meanwhile, the statistical variation in number densities generally increases with burnup. Several possible ways of reducing the statistical errors are discussed: 1) to increase the number of individual Monte Carlo histories; 2) to increase the number of time steps; 3) to run additional independent Monte Carlo depletion cases. Finally, a new Monte Carlo depletion methodology, called the batch depletion method, is proposed, which consists of performing a set of independent Monte Carlo depletions and is thus capable of estimating the overall statistical errors including both the local statistical error and the propagated statistical error.

Key Words: Monte Carlo depletion, statistical bias, CASMO-5, batch depletion method

1. INTRODUCTION

Monte Carlo depletions are practiced routinely nowadays for reactor burnup analysis [1, 2]. The typical way of performing such a Monte Carlo depletion is to couple the Monte Carlo neutron transport solver with the fuel burnup module. The Monte Carlo calculation provides detailed isotopic reaction rates of various types and neutron fluxes in each spatial burnup region, which are passed on to the burnup module for depletion calculations. The updated nuclide number densities are then fed back into the problem for the next-step Monte Carlo transport calculation. There are three components in such a Monte Carlo depletion: the Monte Carlo neutronic solver, the fuel depletion module, and the linkage between the two. As the continuous-energy, three-dimensional Monte Carlo method involves fewer approximations in solving the transport problem compared to the deterministic method, the Monte Carlo depletion results are commonly assumed as the reference higher-order solution to calibrate deterministic methods. Meanwhile, the statistical implications in Monte Carlo depletions are often ignored.

A unique characteristic of the Monte Carlo solution is its associated statistical errors. It is well recognized that a Monte Carlo estimate without an associated statistical error is essentially meaningless. The statistical errors in Monte Carlo solutions are introduced into nuclide number densities via various types of reaction rates for each nuclide in the burnup region. The nuclide number density errors are then propagated through fuel burnup. As the fuel burnup roughly determines the number of fissions that have

occurred in the fuel, the overall amount of actinides and fission products should not be affected by the Monte Carlo statistical errors. Rather, the propagated errors in number densities reflect the difference in the nuclide distributions within the actinide group and the fission product group. For example, if plutonium quantities are over estimated, there will be correspondingly less U-238 in the fuel. In fact, there have been some studies on the issue of the statistical error propagation in Monte Carlo depletions [3, 4]. The major conclusions were: 1) the propagated errors are insignificant compared to the neutron cross section uncertainties; 2) the effect of propagated errors is low up to 60 MWd/kg for depletion calculations of an 8×8 BWR fuel assembly. These results show that the nuclide number density uncertainties due to propagated statistical errors might not be a concern for typical light water reactor analysis.

However, a fundamental question for a Monte Carlo depletion is the correctness of the overall methodology. Since the procedure of a Monte Carlo depletion appears to be reasonable and straightforward, few have seriously challenged the validity of the method. For a Monte Carlo depletion, the level of statistical fluctuations is important, but secondary compared to expectations of the quantities of interest. In this paper, the full statistical implications of Monte Carlo depletions, including both statistical uncertainties and statistical bias, are exploited from the perspective of the deterministic code, i.e., a modified version of the Studsvik lattice physics code, CASMO-5 [5]. With the modified version of CASMO-5, one can produce the original reference solution along with pseudo Monte Carlo depletion results. The statistical implications are investigated by studying the pseudo Monte Carlo depletion results. Note that the statistical bias is defined as the difference between the average of the pseudo Monte Carlo solutions and the reference solution.

Typical PWR lattice problems are used in this exercise. The reference CASMO-5 solution is deemed as the *true* answer of the problem. The pseudo Monte Carlo results are obtained by randomly perturbing the CASMO-5 flux solution after the two-dimensional method-of-characteristic (MOC) transport calculation. The effective one-group cross sections and one-group fluxes are modified according to a prescribed statistical distribution with user-defined standard deviations. The purpose of this statistical perturbation is to mimic the Monte Carlo solution where the results are always subject to certain statistical errors. This perturbation assumes that the CASMO-5 solution is the statistical mean, i.e., the true answer. One can then perform many such independent pseudo Monte Carlo depletions with this modified version of CASMO-5 by varying the random number seeds for each depletion case.

2. ILLUSTRATION OF A SIMPLE ONE-STEP DEPLETION EXAMPLE

Before moving on to further detailed discussions of the CASMO-5 modifications, it is instructive to look at a simple one-step depletion problem in order to illustrate the statistical implications of the Monte Carlo depletion. Consider a single nuclide at the beginning of step with no precursors in a reactor. The initial number density is given as N_0 with no statistical uncertainties. The only removal mechanism is by neutron absorption. The depletion equation is:

$$\frac{dN}{dt} = -\sigma\phi N = -rN \tag{1}$$

where N is the nuclide number density, σ is the effective one-group absorption cross section, ϕ is the one-group neutron flux, and $r = \sigma\phi$ is the single-atom reaction rate. Most depletion methods assume constant reaction rate over the time step. The nuclide number density at the end of the step t is then,

$$N(t) = N_0 e^{-rt} \tag{2}$$

Now consider a Monte Carlo depletion for this simple case. The Monte Carlo solution gives a single-atom reaction rate r sampled from a certain distribution where the mean is r_0 (true answer) and its standard deviation is σ_r. The reference true solution should be

$$N_{ref}(t) = N_0 e^{-r_0 t} = N_0 e^{-\overline{rt}} \tag{3}$$

In contrast, the Monte Carlo depletion solution is

$$N_{MC}(t) = N_0 \overline{e^{-rt}} \tag{4}$$

The statistical bias can be readily defined as:

$$\text{Bias} = \frac{N_{MC}(t)}{N_{ref}(t)} - 1 \tag{5}$$

The bias is obviously not zero. Therefore, strictly speaking, the Monte Carlo depletion procedure yields incorrect results. The variance of a single Monte Carlo depletion can be defined naturally as:

$$\sigma^2_{N_{MC}(t)} = \overline{\left(N_0 e^{-rt} - N_0 \overline{e^{-rt}}\right)^2} \tag{6}$$

The full statistical implications in Monte Carlo depletions include both the statistical bias and the variance. Evaluation of these quantities would require knowledge of the statistical distributions for the single-atom reaction rate. Here, two types of statistical distributions are tried for the evaluation of the bias: the uniform distribution and the Gaussian (normal) distribution. Table I summarizes the statistical results for this simple one-step depletion problem.

Table I. Statistical results for simple one-step depletion problem

	Uniform distribution	Gaussian distribution
Probability density function (pdf)	$f(r) = \begin{cases} 1/(2a), & \lvert r - r_0 \rvert < a \\ 0, & \text{otherwise.} \end{cases}$	$f(r) = \frac{1}{\sigma_r \sqrt{2\pi}} e^{-(r-r_0)^2/(2\sigma_r^2)}$
$\overline{r} = \int r f(r) dr$	r_0	r_0
$\sigma_r^2 = \int (r - \overline{r})^2 f(r) dr$	$\frac{a^2}{3}$	σ_r^2
$N_{MC}(t) = N_0 \overline{e^{-rt}}$	$N_{ref}(t) \frac{e^{at} - e^{-at}}{2at}$	$N_{ref}(t) e^{\sigma_r^2 t^2/2}$
Statistical bias	$\frac{e^{at} - e^{-at}}{2at} - 1$	$e^{\sigma_r^2 t^2/2} - 1$
$\sigma^2_{N_{MC}(t)}$	$N^2_{ref}(t) \left[\frac{e^{2at} - e^{-2at}}{4at} - \left(\frac{e^{at} - e^{-at}}{2at} \right)^2 \right]$	$N^2_{ref}(t) e^{\sigma_r^2 t^2} \left(e^{\sigma_r^2 t^2} - 1 \right)$

In reality, the product of the standard deviation of the single-atom reaction rate and the time step size is very small, i.e., $\sigma_r t \ll 1$. Under this assumption, the asymptotic statistical bias for both distributions can be shown as:

$$\text{Bias} \simeq +\frac{1}{2}(\sigma_r t)^2 \tag{7}$$

One interesting observation is that the sign of the statistical bias shown in Eq. (7) is positive, which means the Monte Carlo depletion solution overestimates the concentration of the nuclide. For a well behaved burnup solver, the balance of overall nuclides would mean the underestimations of downstream product nuclides. As an example, if the single nuclide considered is U-238, the implication would be less plutonium generation with burnup.

In contrast to the statistical bias, the standard deviation of a single Monte Carlo depletion will be:

$$\frac{\sigma_{N_{MC}(t)}}{N_{ref}(t)} \simeq \sigma_r t \tag{8}$$

While Eq. (7) points out that the Monte Carlo depletion is biased, comparison between Eq. (8) and Eq. (7) shows that the bias is a higher-order quantity compared to the statistical standard deviation of the distribution. Therefore, for the simple one-step depletion problem, the typical Monte Carlo depletion method is a valid approach with a small positive bias.

3. CASMO-5 MODELS AND MODIFICATIONS

CASMO-5 [5] is the next-generation Studsvik lattice physics code for LWR fuel assembly burnup calculations. Based on a 586-group neutron library processed from ENDF/B-VII.0 data, it features the method of characteristics (MOC) for the two-dimensional transport calculation. The primary purpose of CASMO-5 usage is to generate homogenized few-group cross sections for 3-D core simulators. In this paper, CASMO-5 has been modified to produce pseudo Monte Carlo depletion results.

At first glance, it would be counter-intuitive for most people since CASMO-5 is a deterministic code. The way of the modification is to externally inject statistical uncertainties into CASMO-5 flux solution so that the behavior of a Monte Carlo depletion can be simulated. This is done before the fuel burnup calculation module. The advantage of using such a deterministic code is the saving in computer time as well as the ability to obtain a reference depletion solution. Three major modifications are implemented in CASMO-5:

- A random number generator module is included based on standard state-of-the-art random number generation methodology.

- Reaction rates including the effective one-group cross sections and the one-group flux after the flux solution are perturbed. Note that the level of standard deviation is an input specified by the user and applied for all quantities.

- The input module to set the random number parameters, e.g., seeds and strides, options to choose statistical distributions, and the desired magnitude of the standard deviations.

The random number generator and the input module are not of interest as those are well established. The statistical perturbations will be our focus. From the 2-D flux solution, one gets effective one-group fission and absorption cross sections as well as the one-group fluxes. Two statistical distributions are considered: the uniform distribution and the Gaussian distribution. The quantities are modified as:

$$x \leftarrow x \cdot (1 + \sigma_x R) \tag{9}$$

where x could be cross sections or fluxes, σ_x is the user-specified standard deviation in relative units, and R represents the distribution with a mean of zero and a standard deviation of unity. For simplicity, a constant standard deviation is assumed and applied for all quantities. In real Monte Carlo calculations, the standard deviation will depend on spatial burnup regions, nuclides and reaction types. Furthermore, statistical correlations exist among certain reaction types.

With this modified version of CASMO-5, one will be able to model the Monte Carlo depletion. For a given depletion problem, one can run the CASMO-5 depletions many times using a distinct random number sequence for each depletion. Each CASMO-5 output represents a single Monte Carlo depletion, and the statistical implications can be studied by examining the set of these independent Monte Carlo depletions. The unperturbed CASMO-5 depletion solution is used as the reference solution.

4. RESULTS AND DISCUSSIONS

4.1. Pin Cell Depletion

A typical 4.5 w/o enriched UO_2 pin cell problem is examined. The reference depletion is the default CASMO-5 solution without modifications. A set of 500 Monte Carlo depletions with a 1% standard deviation in one-group cross sections and fluxes is run, each of which has a different random number sequence. The nuclide number densities and eigenvalues with burnup will be investigated, and both uniform and Gaussian distributions are exercised.

Figs. 1 and 2 are from the 1% standard deviation of uniform distributions. Figure 1 illustrates the dispersion of eigenvalue differences from the reference solution for the set of Monte Carlo depletions. The depletions spread over a range of few hundred pcm's in eigenvalue space. A trend of increasing eigenvalue uncertainty with burnup can be seen due to the propagated statistical uncertainties. At the beginning of life, there are no burnup-related statistical uncertainties for the fresh fuel. However, the equilibrium xenon in the first depletion step causes a step change of the eigenvalue uncertainty from exactly zero to a finite value.

Figure 2 shows the U-235 number density histogram at three burnup levels. As burnup increases, the statistical error of U-235 number density increases. This agrees with general perceptions that the propagated statistical errors should increase with time as the Monte Carlo statistical errors are continuously injected into the depletion.

Figure 3 shows the statistical information for eigenvalue. The statistical bias is negligible compared to the standard deviation. Note that these standard deviations reflect only the propagated error in number densities. Since CASMO-5 is a deterministic code, there is no statistical error in the flux solution. A real Monte Carlo depletion will have a larger standard deviation due to the local statistical error as well. Another interesting observation is that the types of statistical distributions used have no effect on the final results.

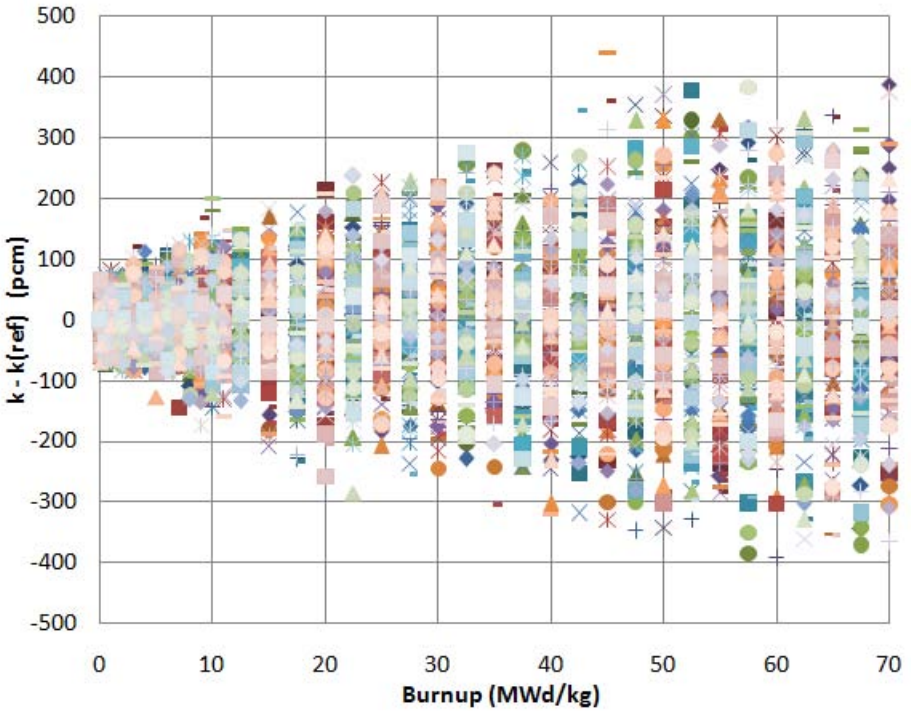


Figure 1. Illustration of Monte Carlo depletions.

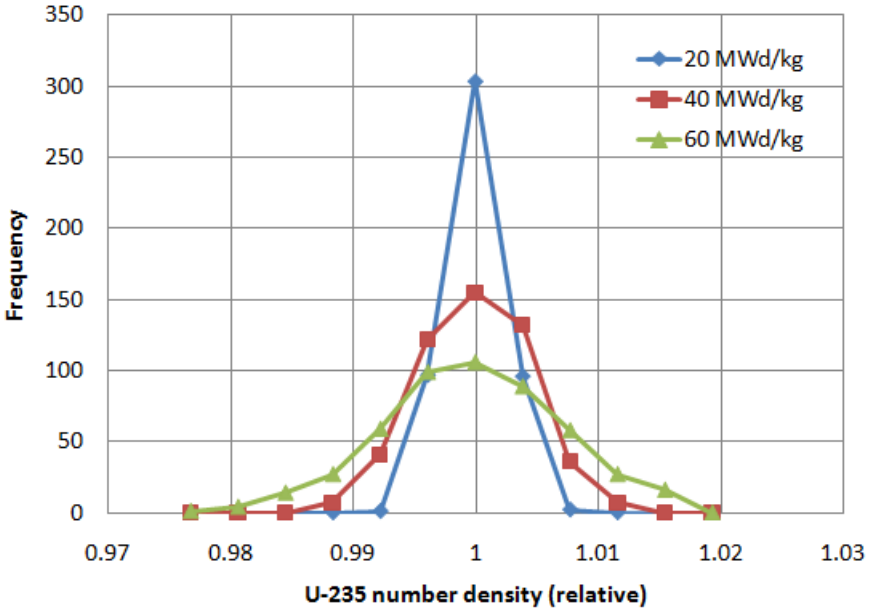


Figure 2. U-235 number density distribution for 1% uniform distribution.

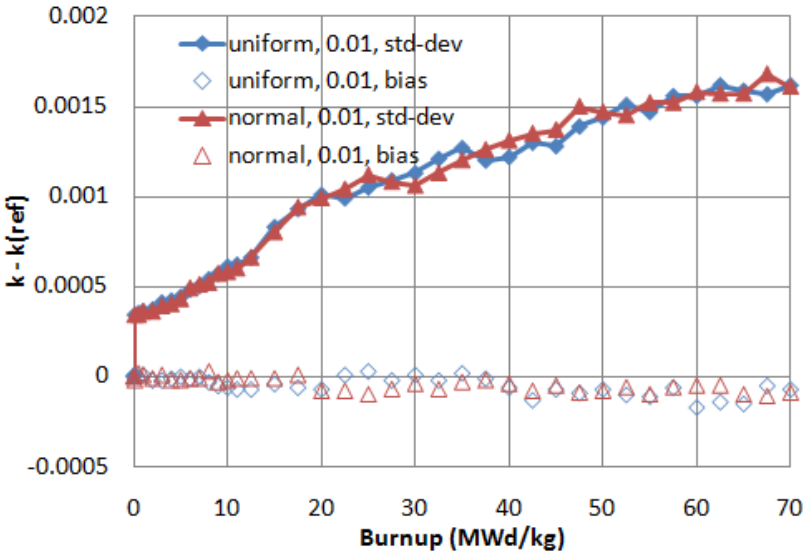
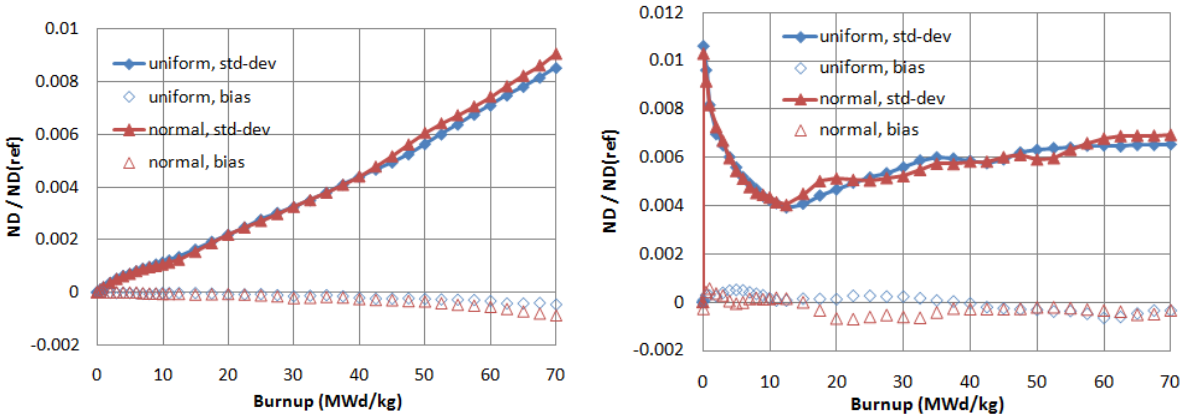
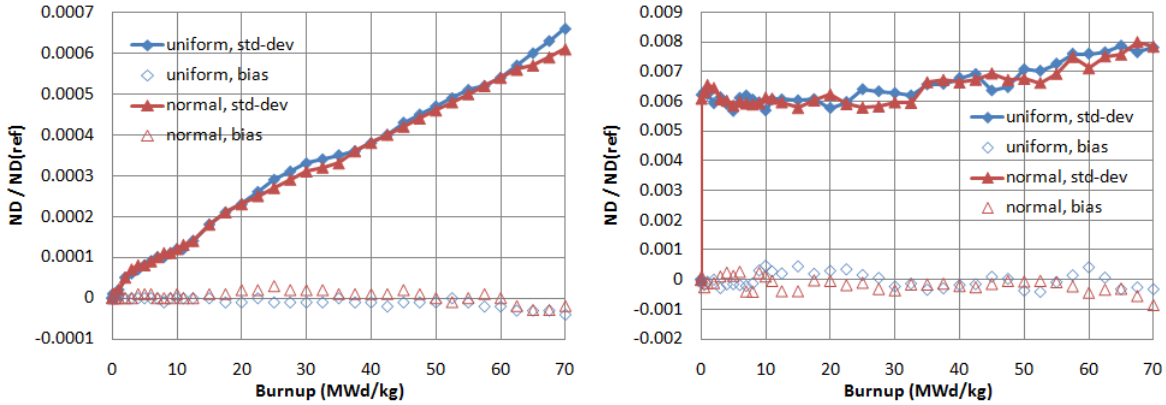


Figure 3. Statistical uncertainties in eigenvalue.



(a) U-235 (b) Pu-239



(c) Tc-99 (d) Xe-135

Figure 4. Statistical uncertainties in various number densities.

Figure 4 further shows statistical uncertainties in a few nuclide number densities. For those equilibrium isotopes depending on flux level, the standard deviation is almost independent of the burnup. Physically, the concentrations of those nuclides do not change significantly over burnup. Most other nuclides are either asymptotically decreasing, such as U-235, or accumulating, such as stable fission products. Their statistical errors grow almost linearly with burnup. For all cases investigated, the statistical bias is significantly smaller compared to the standard deviation.

Table II. Summary of statistical results for pincell problem with 1% uniform distribution

	0.1 MWd/kg		20 MWd/kg		40 MWd/kg		60 MWd/kg	
	Std	Bias	Std	Bias	Std	Bias	Std	Bias
K-inf (pcm)	34	+1	101	–7	122	–6	156	–17
U-234 (%)	0.00	+0.00	0.09	–0.00	0.16	–0.00	0.23	–0.01
U-235 (%)	0.00	+0.00	0.22	–0.01	0.44	–0.02	0.71	–0.03
U-236 (%)	5.29	+0.29	1.82	+0.11	1.37	+0.11	1.14	+0.11
U-238 (%)	0.00	+0.00	0.00	–0.00	0.01	–0.00	0.01	–0.00
Np-237 (%)	5.82	+0.34	1.37	+0.07	1.12	+0.09	0.94	+0.08
Pu-238 (%)	0.00	+0.00	1.50	+0.06	1.22	+0.10	0.99	+0.09
Pu-239 (%)	1.06	+0.01	0.47	+0.01	0.59	–0.00	0.65	–0.06
Pu-240 (%)	2.04	+0.10	1.53	+0.03	1.32	+0.10	1.30	+0.04
Pu-241 (%)	2.58	+0.10	0.99	+0.05	0.85	+0.02	0.79	+0.02
Pu-242 (%)	0.00	+0.00	2.21	+0.11	1.76	–0.01	1.38	+0.02
Am-241 (%)	0.00	+0.00	0.89	+0.09	0.82	+0.01	0.85	+0.01
Am242m (%)	0.00	+0.00	1.14	+0.08	1.10	+0.01	1.19	+0.04
Mo-95 (%)	0.01	+0.00	0.03	–0.00	0.04	–0.01	0.05	–0.01
Tc-99 (%)	0.00	+0.00	0.02	–0.00	0.04	–0.00	0.05	–0.00
Rh-103 (%)	0.05	–0.00	0.11	+0.01	0.15	–0.01	0.18	–0.00
Cs-133 (%)	0.00	+0.00	0.03	–0.00	0.06	–0.00	0.07	–0.01
Xe-131 (%)	0.01	–0.00	0.08	–0.00	0.14	+0.00	0.19	–0.01
Xe-135 (%)	0.62	–0.02	0.58	+0.03	0.68	–0.02	0.76	+0.04
Nd-143 (%)	0.01	+0.00	0.08	–0.00	0.12	–0.00	0.19	–0.01
Nd-145 (%)	0.00	+0.00	0.04	–0.00	0.06	–0.00	0.08	–0.01
Pm-147 (%)	0.01	–0.00	0.13	–0.01	0.18	–0.00	0.23	–0.00
Sm-149 (%)	0.09	–0.00	0.84	+0.04	0.89	–0.06	0.92	–0.01
Sm-151 (%)	0.04	–0.00	0.60	–0.04	0.74	–0.04	0.74	+0.01
Sm-152 (%)	0.05	–0.00	0.20	–0.01	0.27	–0.00	0.34	–0.01
Eu-153 (%)	0.07	–0.00	0.26	+0.03	0.22	+0.00	0.24	–0.00

In conclusion, the results of the pin cell depletion are consistent with the analytic results for the simple one-step depletion problem. Table II summarizes all findings and no numerical bias is observed. This explains the well-known self-correcting behavior of Monte Carlo depletion. The reason behind this is that the bias is small and overwhelmed by the standard deviations. Furthermore, the propagated depletion error increases with burnup, but the fluctuation level is small compared to the imposed 1% reaction rate standard deviation up to 60 MWd/kg for important nuclides.

4.2. Extending To Fuel Lattice Depletion

The next step in LWR burnup calculation is to extend the above pin cell depletion to the fuel lattice depletion. One important difference is that there are now multiple burnup regions. Thus, the perturbation of the flux implies the perturbation in the local power. On the other hand, if based on the same number of neutron histories, the real Monte Carlo depletion will have worse statistics for individual burnup region compared to the pin cell depletion because the tally scores are reduced. The statistical error of the global parameters would be comparable to pin cell depletion as those are integrated over the entire problem.

As a test to understand the statistical uncertainties for fuel lattice calculation, one could consider the previous pin cell depletion and a new 5×5 uniform lattice depletion. The 5×5 lattice problem is constructed using the exactly same pin cell as used in the previous pin cell problem. As the type of statistical distribution is shown to be irrelevant in the pin cell problem, only uniform distribution is used. The previous pin cell depletion has a standard deviation of 1% for cross sections and fluxes. If using the same number of neutron histories for the 5×5 lattice depletion, the standard deviation for cross sections and fluxes will be 5% for individual rod in the 5×5 lattice since the neutron histories allocated for each rod is reduced by a factor of 25. Correspondingly, a set of 500 Monte Carlo depletions is performed for the 5×5 lattice with a 5% standard deviation for cross sections and fluxes in individual rods.

Figure 5 shows the comparison of statistical uncertainties in eigenvalue between the previous pin cell depletion and the 5×5 lattice depletion. The statistical uncertainties agree very well confirming the above analysis of the statistical uncertainties.

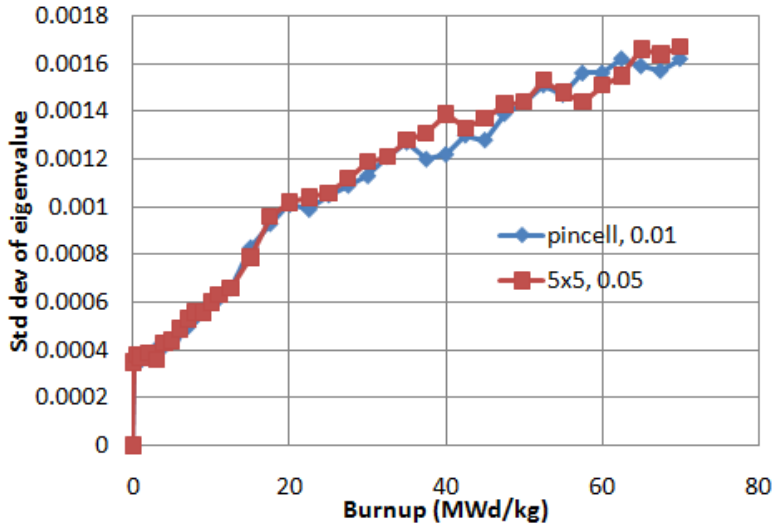


Figure 5. Comparison of statistical uncertainties between pin cell and 5×5 lattice.

One issue for the Monte Carlo fuel lattice depletion is the ability to accurately predict the power distribution within the bundle during burnup. Conceptually, the above 5×5 uniform lattice should have a peaking factor of unity independent of the burnup. However, the peaking factor at 60 MWd/kg is found to be around 1.03; and the location of the peak rod changes depending on the specific random number sequence used. This will be a challenge for the Monte Carlo depletion method as the detailed spatial power distributions are needed with burnup for the reactor analysis.

The statistical error of a real Monte Carlo calculation for the assembly is indeed much smaller than the assumed 1% standard deviation for the pin cell problem in this paper. Typical reported standard statistical errors for a BWR 10×10 lattice with one million neutron histories are: 60 pcm of eigenvalue, 0.6% of pin fission rate, and 0.3% of neutron flux in a fuel rod [1]. The equivalent pin cell problem will have about a factor of 10 smaller statistical error, which corresponds to ~0.05% distribution (note that 1% distribution results are included in this paper). Therefore, in order to estimate the error in real Monte Carlo depletions, one can do so by dividing the reported results by a factor of ~20. It is then obvious that the accumulated statistical errors (eigenvalue and nuclide number densities) in real Monte Carlo depletions can be safely ignored. Nevertheless, it is reassuring if the Monte Carlo depletions could provide total error bars for neutronic properties as well as isotopic number densities.

4.3. Reducing Statistical Uncertainties

The statistical error in the Monte Carlo depletion can be reduced in various ways: 1) to increase the number of individual Monte Carlo histories; 2) to increase the number of depletion time steps; 3) to run additional independent Monte Carlo depletion cases.

The first approach of increasing the neutron histories is the most straightforward one. In general, the statistical error in the Monte Carlo solution decreases as inversely proportional to the square root of the number of histories. This also applies to the propagated statistical error. One can verify this by varying the magnitude of the standard deviation in the pin cell depletion problem. It is found that the statistical uncertainties in the Monte Carlo depletion are indeed proportional to the user-defined standard deviation.

The second approach to reduce the depletion statistical uncertainties is to increase the number of time steps for the problem. Ref. [4] shows that the deviation in number densities would be improved by a factor of $1/\sqrt{k}$ if the number of depletion steps increases by a factor of k. There are additional benefits from the increasing the number of depletion steps, i.e., the depletion truncation error will be minimized with finer steps.

The third approach is to run additional independent Monte Carlo depletion cases by using different random number sequences. It reduces the statistical uncertainties by a factor of $\sqrt{n}$ where n is the total number of Monte Carlo depletions.

In summary, in terms of reducing the Monte Carlo depletion uncertainties, all three approaches have the same convergence rate, i.e., $1/\sqrt{T}$, where T is the computer time.

5. MONTE CARLO BATCH DEPLETION METHOD

The existing Monte Carlo depletion method is based on a single depletion where the statistical errors in the burnt fuel number densities are not estimated. There are at least two ways to estimate Monte Carlo depletion errors: 1) to explicitly compute the error propagation for each burnup step, and stick to a single Monte Carlo depletion; 2) to do a set of Monte Carlo depletions for the same problem (called batch depletion method).

The first approach is essentially a perturbation approach. For a time step between 0 and t, the depletion solution in the matrix exponential form will be:

$$\mathbf{x}(t) = e^{\mathbf{A}t}\mathbf{x}(0) \tag{10}$$

where **x** is the isotopic vector containing the nuclide quantities and **A** is the transition matrix containing all rate constants (such as single-atom reaction rate and decay constants). The end of step number density uncertainties come from either the beginning of step number densities or the transition matrix. The two contributions can be combined to get the end of step number density uncertainties. Detailed discussions can be found in Ref. [3].

The second approach is named the batch depletion method in this paper. It is a much more straightforward approach. Instead of running a big depletion job, one can divide this problem into a set of independent fast depletions and combine the results at each burnup point. By doing so, the number density statistical errors can be estimated based on the set of Monte Carlo depletions. In fact, all other neutronic parameters could be computed similarly. For example, the eigenvalue will also be averaged across the set of depletions. The local statistical error should be smaller than the overall uncertainty that includes the propagated error.

Figure 6 shows the schematics of performing the Monte Carlo batch depletion. For a given depletion problem, a set of n Monte Carlo depletions are initiated. At each burnup point, the Monte Carlo depletion results are the average of these n Monte Carlo results. In this scheme, instead of just storing one number density for each nuclide in the problem, there will be n values to keep track of each individual Monte Carlo depletion case. The random number sequence needs to be explicitly set and distinct for each case.

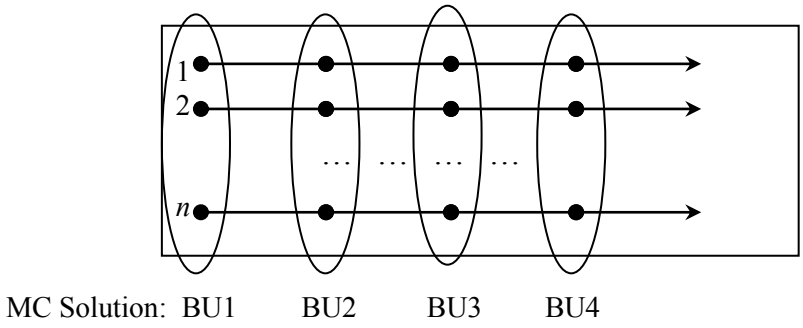


Figure 6. Schematic of the Monte Carlo batch depletion method.

One issue is the fission source convergence problem arising from high dominance ratios. However, the Monte Carlo application here is limited to the assembly level calculations, e.g., few-group cross section generations, and this issue is not a real concern.

In summary, the batch depletion method is capable of estimating the overall statistical errors including both the local statistical error and the propagated statistical error. It is worthwhile in the future to expand the current single Monte Carlo depletion methodology to include the batch depletion methodology.

6. CONCLUSIONS

In this paper, the statistical implications of Monte Carlo depletions are investigated. As shown in the analytical results for the simple depletion case and demonstrated by the CASMO-5's pseudo Monte Carlo results, the statistical bias is proved to be negligible compared to the standard deviation. Therefore, the Monte Carlo depletion approach is a statistically correct approach for burnup calculations.

The statistical uncertainties in the number densities generally grow with burnup. This could be a concern for high burnup applications. Various ways of reducing the depletion statistical error are discussed, all of which have the same convergence rate.

As a way to estimate the number density errors, the Monte Carlo batch depletion method is proposed as a promising approach for future developments.

ACKNOWLEDGMENTS

This primary author wishes to thank Dr. Scott Palmtag for the initial suggestions on understanding the statistical implications of Monte Carlo depletions using CASMO.

REFERENCES

1. Z. Xu, J. Rhodes, K. S. Smith and N. Gheorghiu, "MCNP-5/ORIGEN-2.2/MCODE-2.2 versus CASMO-5 Depletion for A Heavy Gd-Poisoned BWR Fuel Assembly," *Trans. Am. Nucl. Soc.*, **96**, pp.575-577 (2007).
2. A. Glaser, "Isotopic Signatures of Weapon-Grade Plutonium from Dedicated Natural Uranium-Fueled Production Reactors and Their Relevance for Nuclear Forensic Analysis," *Nucl. Sci. Eng.*, **163**, pp.26-33 (2009).
3. T. Takeda, N. Hirokawa and T. Noda, "Estimation of Error Propagation in Monte Carlo Burnup Calculations," *J. Nucl. Sci. Technol.*, **36**, pp.738-745 (1999).
4. M. Tohjoh, T. Endo, M. Watanabe and A. Yamamoto, "Effect of Error Propagation of Nuclide Number Densities on Monte Carlo Burnup Calculations," *Ann. Nucl. Energy*, **33**, pp.1424-1436 (2006).
5. J. Rhodes, K. Smith and D. Lee, "CASMO-5 Development and Applications," *PHYSOR-2006, ANS Topical Meeting on Reactor Physics*, Vancouver, BC, Canada (2006).