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QUADRATIC DEPLETION MODEL FOR GADOLINIUM ISOTOPES IN CASMO-5

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ABSTRACT

The huge absorption cross sections of Gd-155 and Gd-157 cause strong spatial shielding effects in Gd-bearing pins. A quadratic depletion model has been implemented in CASMO-5 in order to address the issue of small depletion steps typically required for Gd-bearing fuel assemblies. In this model, the microscopic absorption reaction rates of gadolinium isotopes are assumed to be quadratic functions of the number density of Gd-155. This quadratic function assumption models the variations of the spatial shielding effects over the depletion step and therefore improves the overall accuracy of depletion calculations. With this new model, a depletion step size four times larger than the step size for a conventional predictor-corrector method can be used for Gd-bearing assemblies without compromising accuracy.

1. INTRODUCTION

CASMO-5 is the latest version of Studsvik's lattice physics code.[1] A summary of new features in CASMO-5 is as follows:

- 586 group neutron data library based on ENDF/B-VII
- 18 group gamma data library based on ENDF/B-VII
- 586 energy group pincell calculation
- More detailed geometries in method of characteristics (MOC) calculation, e.g., BWR cruciform control rods
- Higher order P_n scattering capability
- Characteristics based Dancoff calculations
- Multi-assembly calculation capability
- Spent nuclear fuel data generation
- Expanded depletion chains
- U-238 upscatter model[2]
- Explicit energy release per fission model[3]

- Quadratic depletion for gadolinium isotopes

This paper presents the details of the quadratic depletion model for gadolinium isotopes.

For the general depletion calculation of typical LWR lattices, CASMO uses a conventional predictor-corrector (PC) method with a depletion step size of 1.0 MWd/kgU.[4,5] However, for the depletion of Gd-bearing fuel assemblies, the default depletion step size between 2D MOC calculations is 0.5 MWd/kgU. In each step, CASMO-4 employs 4 substeps, each with a step size of 0.125 MWd/kgU, in which CASMO-4 performs the resonance calculation, pincell calculation, and burnup calculation but omits the expensive 2D transport solution. With this substepping method, CASMO-4 permits a depletion step size two times larger than the conventional PC method.

The newer CASMO-5 code employs a higher-order depletion method rather than multiple small substeps with a low order depletion. By assuming a quadratic shape of the microscopic reaction rates over the corrector step, CASMO-5 can avoid the cumbersome substepping and still improve the accuracy of the depletion calculation for the Gd-bearing assemblies.

2. QUADRATIC DEPLETION MODEL

Considering only neutron capture, as can be done for gadolinium isotopes, a burnup equation for isotope, m, can be written as

$$\frac{dN_m(t)}{dt} = N_{m-1}(t)\sigma_{m-1}(t)\varphi(t) - N_m(t)\sigma_m(t)\varphi(t), \qquad (1)$$

where $N_m(t)$ is the number density of isotope, m, as a function of time, t, $\sigma_m(t)$ is the microscopic capture cross section of isotope, m, and $\varphi(t)$ is neutron flux. In the conventional PC method, the absorption reaction rate, $\sigma_m(t)\varphi(t)$, is assumed to be constant. This assumption is a good assumption for small depletion step sizes. Even for practically reasonable depletion step sizes, combined with two depletion calculations (predictor & corrector), the PC method gives accurate solutions for normal LWR assemblies in the absence of gadolinium. However, if gadolinium is present, the depletion step size must be reduced to maintain accuracy.

A conventional PC method is applied to deplete a BWR GE14 assembly with 10x10 pins, shown in Fig. 1. Red pins represent 7% Gd_2O_3 pins. Fig. 2 shows k-infinity through the depletion. Usually the step size of 1.0 MWd/kgU is sufficiently fine for typical LWR assemblies; however, it is apparent that the depletion step of 1.0 MWd/kgU is not fine enough for this Gd-bearing assembly. Eigenvalues at a burnup of 18.0 MWd/kgU are summarized in Table 1. It is easily noted that, in case of Gd-bearing assemblies, the PC method requires smaller depletion steps by a factor of 10 to retain comparable accuracy to non-Gd assemblies.

In order to investigate the need for fine depletion steps in Gd-bearing assemblies, the microscopic capture reaction rates of Gd-155 and Gd-157 are plotted in Fig. 3. The shape is varying non-linear and the magnitude varies by several orders of magnitude over the depletion. This significant variation of the absorption reaction rates is caused by spatial shielding effects of gadolinium isotopes. Gd-155 and Gd-157 have huge capture cross sections in the thermal neutron energy range and burn out quickly as shown in Fig. 4. Unless very fine depletion steps are used (at least initially), it is difficult to obtain good accuracy with the conventional PC method.

The spatial shielding effect is a function of the isotopic number density and therefore the capture reaction rates are plotted in Fig. 5 as a function of the number density of Gd-155 which has a large capture cross section and persists longer than Gd-157. It can be easily noted that the reaction rates are smooth functions of Gd-155 number density and the dependency is non-linear.

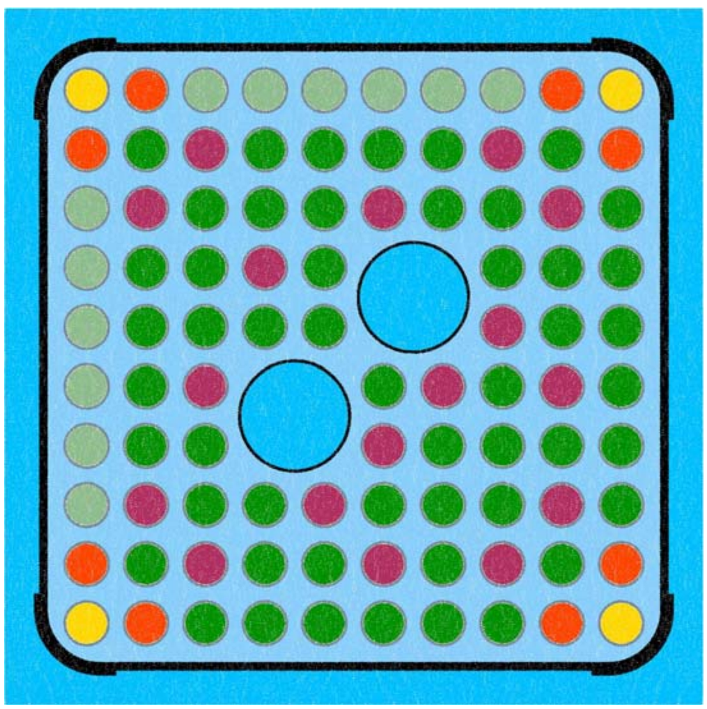

Fig. 1 GE14 Assembly with 17 Gd (7%) Pins

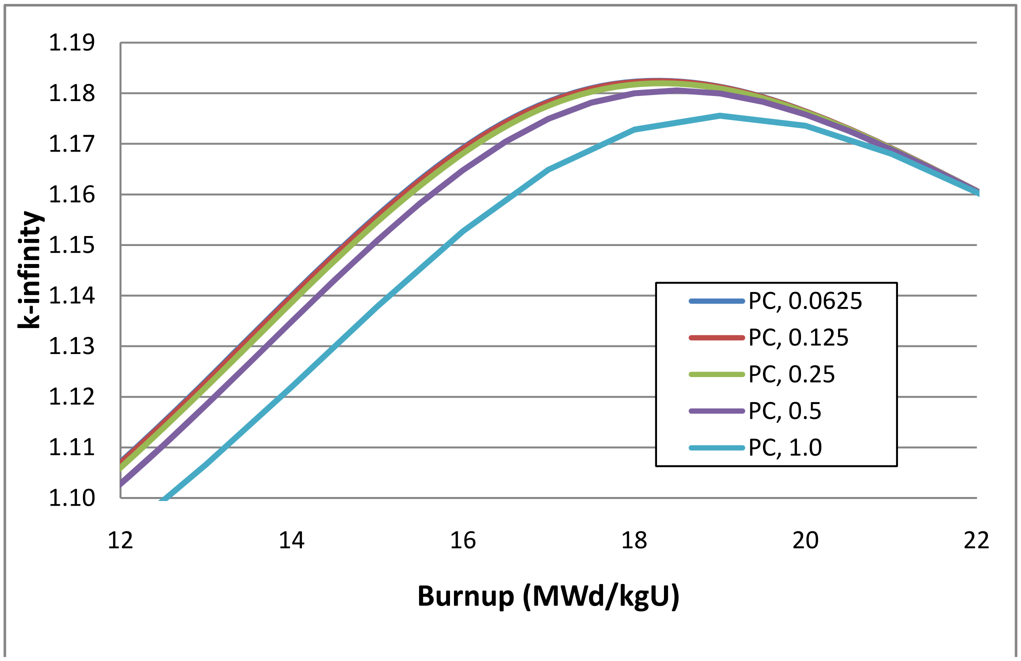

Fig. 2 K-inf vs. Burnup for GE14 Assembly with 17 Gd pins

Table 1. Impact of Gd Pins on Accuracy of GE14 Assembly Burnup Calculation (18.0 MWd/kgU)

Step Size (MWd/kgU)	No Gd		17 Gd Pins	
	k-inf	Diff (pcm)	k-inf	Diff (pcm)
0.0625	1.20939	-	1.18231	-
0.125	1.20938	-1	1.18218	-9
0.25	1.20935	-3	1.18174	-41
0.5	1.20930	-6	1.18000	-166
1.0	1.20921	-12	1.17283	-684

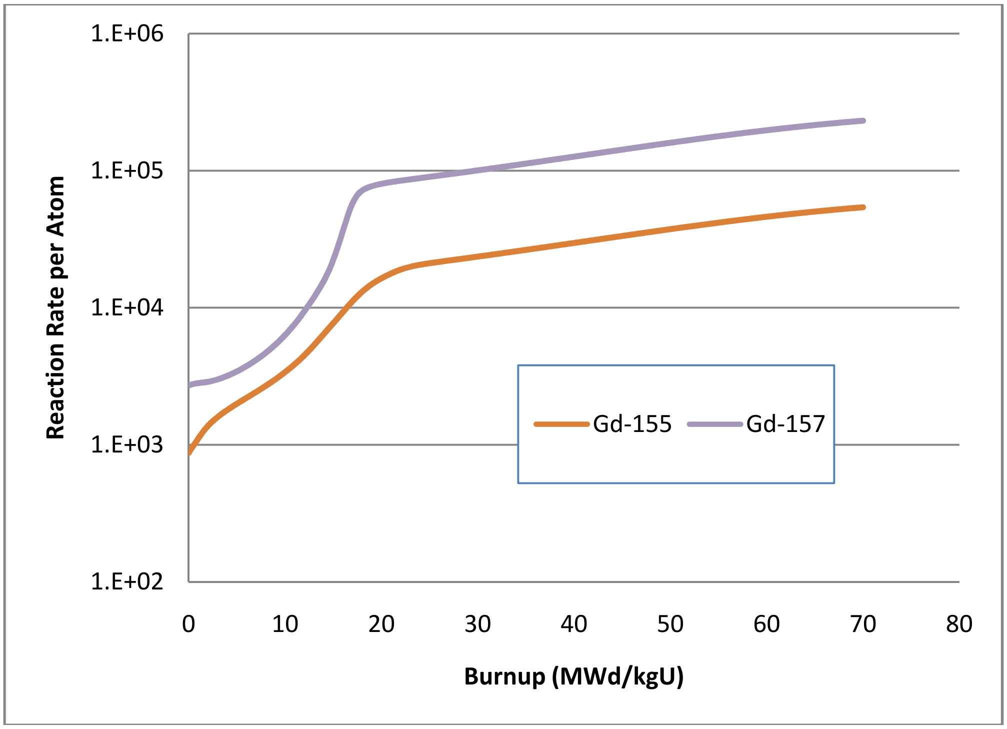

Fig. 3 Microscopic Capture Reaction Rates of Gd-155 and Gd-177 (vs. Burnup)

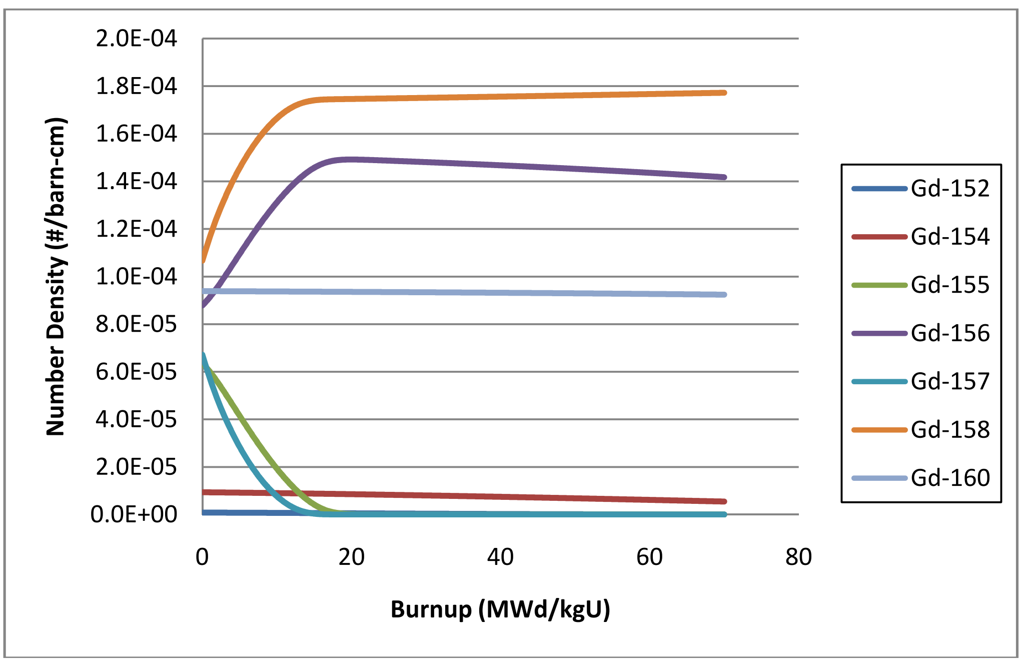

Fig. 4 Number Densities of Gadolinium Isotopes

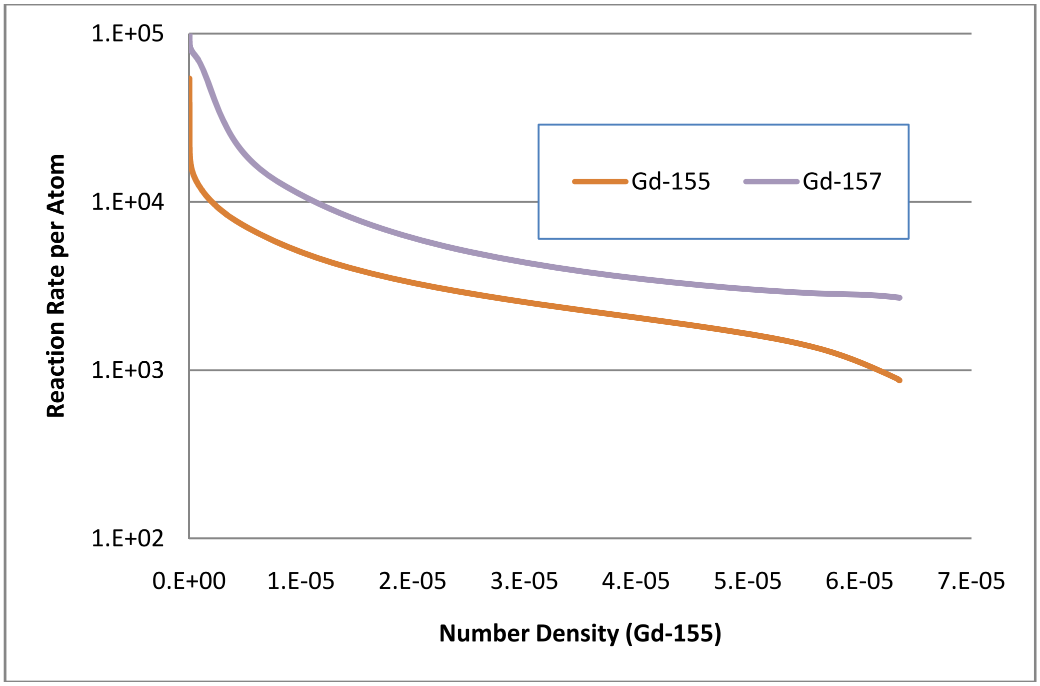

Fig. 5 Microscopic Capture Reaction Rates of Gd-155 and Gd-177

(vs. Gd-155 Number Density)

Recently, Yamamoto et al. proposed a projected predictor-corrector method in order to improve the accuracy of the depletion calculation of Gd-bearing assemblies.[6] Yamamoto's method is based on the assumption that the microscopic absorption reaction rate is a linear function of its number density.

In this paper, a quadratic depletion (QD) method is presented, in which the microscopic capture reaction rates of gadolinium isotopes are assumed to be quadratic functions of Gd-155 number density. This QD method has been used for the last 5 years in CASMO-4E[7] and is now the default depletion model for gadolinium isotopes used in CASMO-5. The quadratic assumption can be written as:

$$R_m \equiv \sigma_m \varphi = a_m \Delta N_3^2 + b_m \Delta N_3 + c_m \ , \quad (2)$$

where R_m is the reaction rate per atom of isotope m, a_m, b_m, and c_m are coefficients of quadratic function, $\Delta N_3 = N_3(t) - N_3(t_n)$, and m = 1, …, 7 for Gd-152, Gd-154, Gd-155, Gd-156, Gd-157, Gd-158, and Gd-160.

Three coefficients for each gadolinium isotopes can be determined using three sets of $(R_m, \Delta N_3)$. For the corrector step from t_n to t_{n+1} (where n is depletion step number), this data set is known at t_{n-1}, t_n, and t_{n+1}. For t_{n+1}, the predicted information from a conventional PC method is used, i.e., the QD method is applied to corrector step only.

Once all the coefficients are determined, Eq. (2) is inserted into Eq. (1). Then one can solve Eq. (1) easily by numerical integration from t_n to t_{n+1}. In this QD method, the predictor number densities and reaction rates are used just to determine the coefficients of the quadratic functions, and they are not necessarily accurate due to the fact that it still assumes constant microscopic reaction rates. However, the corrector step number densities from the quadratic depletion are very accurate as will be shown in the next section.

Usually in the PC method (QD method too), only one flux calculation is performed with predictor step number densities to save calculation time. In this case, even though the corrector step number densities are accurate, the flux and eigenvalue solutions may not be as accurate as the corrector number densities. One way to overcome this deficiency is the post correction of the predictor number densities:

$$N_{m,n+1}^{p,u} = N_{m,n}^{c} \exp \left[\ln\left(N_{m,n+1}^{p} / N_{m,n}^{c} \right) * f_{m,n}\right] \ , \tag{3}$$

where the correction factor is defined as

$$f_{m,n} = f_{m,n-1} * \frac{\ln\left(N_{m,n}^{c} / N_{m,n-1}^{c} \right)}{\ln\left(N_{m,n}^{p} / N_{m,n-1}^{c} \right)} \ , \tag{4}$$

where subscripts m, n are indexes for isotope and time step, respectively, superscripts p, c, u stand for predictor, corrector, and post-corrected. This post-correction is based on the approximation that the error in the number density ratio is constant over the depletion. After the post-correction, the flux calculation is performed with the corrected predictor step number density, $N_{m,n+1}^{p,u}$. This post-correction is very important for the accuracy of QD method. Numerical tests show that more than 50% of the accuracy improvement comes from this post-correction.

One key assumption for QD is that the reaction rates vary smoothly with depletion which makes it well suited for a constant condition depletion as is typically performed in a lattice physics code.

3. NUMERICAL RESULTS

The QD method has been applied to a GE14 assembly with 17 Gd_2O_3 pins. Post-correction factors are plotted in Fig. 6. At the first depletion step, these factors are set to 1.0 and vary smoothly over the depletion. Once the number density reduces below 1.e-6 /barn-cm, then the factors are reset to 1.0.

Eigenvalues are plotted as a function of burnup in Fig. 7. The case with a depletion step size 0.0625 MWd/kgU with the PC method is used as the reference. It is obvious that there is a huge improvement using QD compared to the standard PC method.

Fig. 8 shows the eigenvalue behavior at a burnup of 18 MWd/kgU as a function of depletion step size for PC and QD (with and without post-correction) methods. The QD method is insensitive to the depletion step size and, with a step size of 0.5 MWd/kgU, it shows an almost converged solution in terms of depletion step size.

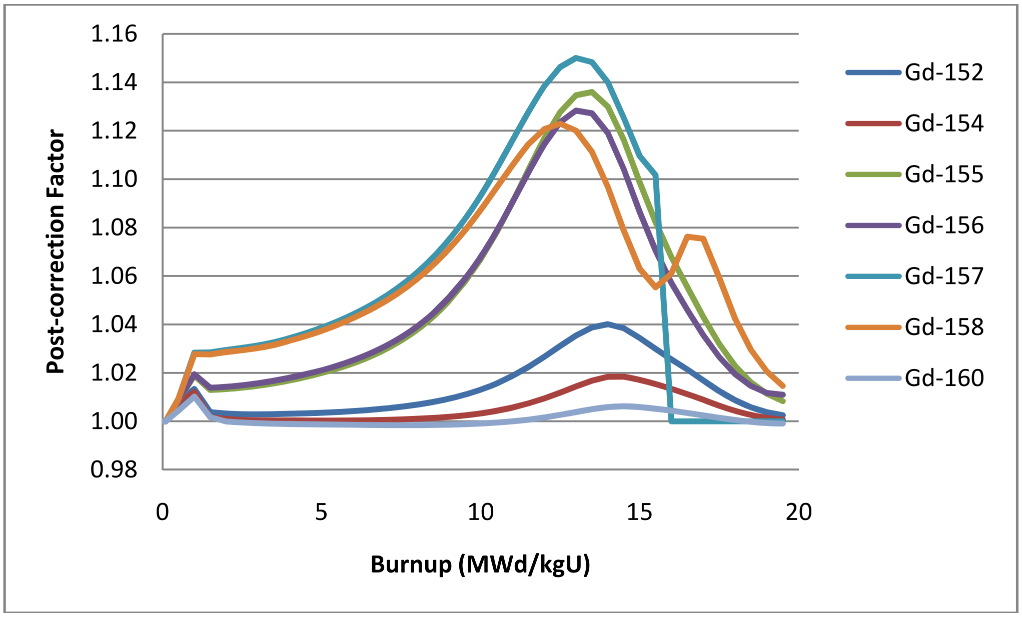

Fig. 6 Post-correction Factors for Predictor Number Densities of Gadolinium Isotopes (GE14, QD, 0.5)

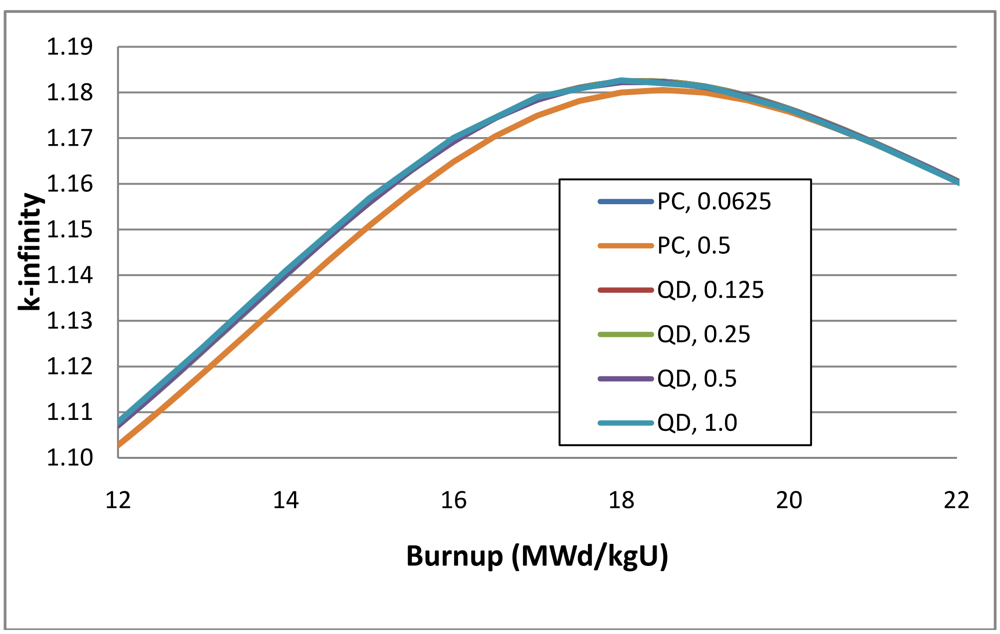

Fig. 7 k-infinity vs. Burnup for GE14 Assembly with 17 Gd Pins (QD method)

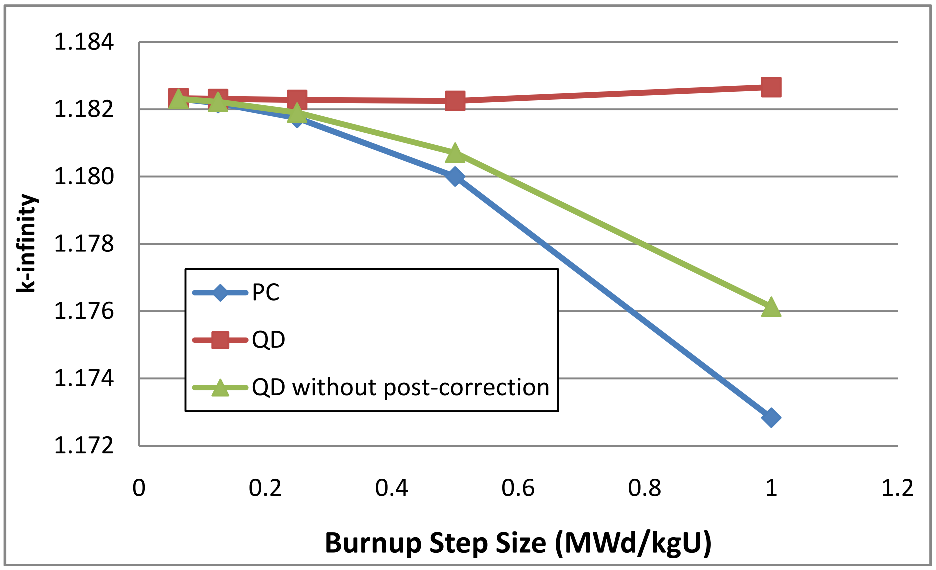

Fig. 8 k-infinity of GE14 Assembly with 17 Gd Pins at 18.0 MWd/kgU

4. CONCLUSION

The root cause of the necessity of small depletion steps for Gd-bearing assembly has been identified as the strong spatial self-shielding of Gd-155 and Gd-157, and a quadratic depletion model has been proposed to overcome this difficulty and to increase the depletion step size without compromising the accuracy of depletion calculation. The newly proposed method has been implemented in CASMO-5 and test results show significant improvement in accuracy for larger depletion step sizes with a GE14 assembly containing Gd_2O_3 pins. It was observed that QD method with the step size of 0.5 MWd/kgU produces very accurate results for Gd-bearing fuel assemblies.

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